OLIMPIADA NAŢIONALĂ DE MATEMATICĂ
Etapa locală - 14.02.2015
Clasa a VII-a
Bareme de corectare şi notare

1. a) $\left(1>\frac{1}{\sqrt{2015}},\ \frac{1}{\sqrt{2}}>\frac{1}{\sqrt{2015}},\frac{1}{\sqrt{3}}>\frac{1}{\sqrt{2015}}\ ,\ldots,\ \frac{1}{\sqrt{2015}}\geq\frac{1}{\sqrt{2015}}\right)$ **(1p)**

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{2015}}>\underbrace{\frac{1}{\sqrt{2015}}+\frac{1}{\sqrt{2015}}+\ldots+\frac{1}{\sqrt{2015}}}_{2015}$$ **(1p)**

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{2015}}>\sqrt{2015}$$ **(1p)**

b) $x=\frac{\sqrt{2}}{\sqrt{2}}-\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{6}}-\frac{\sqrt{2}}{\sqrt{6}}+\ldots+\frac{\sqrt{2015}}{\sqrt{2015\cdot2014}}-\frac{\sqrt{2014}}{\sqrt{2015\cdot2014}}$ **(1p)**

$$x=\frac{\sqrt{2015}-1}{\sqrt{2015}}$$ **(1p)**

$$1936<2015<2025\Leftrightarrow 44<\sqrt{2015}<45\Big|_{-1}\Leftrightarrow 43<\sqrt{2015}-1<44$$ **(1p)**

$$\left[\sqrt{2015}-1\right]=43\Leftrightarrow\left[x\sqrt{2015}\right]=43$$ **(1p)**

2. a) $\sqrt{a^2+b^2+c^2}=\sqrt{2c^2}=c\sqrt{2}$ **(1p)**

$c\sqrt{2}\in\mathbb{Q}\Rightarrow\exists r\in\mathbb{Q}$ astfel încât $c\sqrt{2}=r\Leftrightarrow\underset{\notin\mathbb{Q}}{\sqrt{2}}=\underset{\in\mathbb{Q}}{\frac{r}{c}}$ Contradicţie! $\sqrt{a^2+b^2+c^2}\notin\mathbb{Q}$ **(1p)**

b) $a^2,b^2\in\{5k+1;5k+4/k\in\mathbb{N}\}$ **(1p)**

a^2,b^2 au ambele aceeaşi formă, *5k+1* sau *5k+4*, atunci avem contradicţie **(1p)**

a^2,b^2 au forme diferite, $a^2+b^2=5q\Rightarrow c^2=5q\Rightarrow c\vdots5\Leftrightarrow c^2=25p^2\Leftrightarrow a^2+b^2=25p^2$ **(1p)**

Putem considera $a^2=9p^2$ şi $b^2=16p^2$ **(1p)**

$(a,b)=1\Rightarrow p=1$. Avem $a=3$, $b=4$ şi $c=5$. $\sqrt{a^2b^2+c^2}=\sqrt{9\cdot16+25}=\sqrt{169}=13\in\mathbb{N}$ **(1p)**

3. a) $(DN) \equiv (BM)$ **(2p)**

b) $(DM) \equiv (BN)$ **(1p)**

DMBN paralelogram **(1p)**

DLBT paralelogram **(2p)**

O este mijlocul lui $(LT) \Rightarrow$ *L*, *O*, *T* sunt coliniare **(1p)**

4. $(BE) \equiv (CE)$ **(2p)**

$\frac{AE}{AC} = \frac{BD}{BC} = \frac{1}{3} \Rightarrow DE \parallel AB$ **(1p)**

Prin *O*, ducem $MN \parallel AB, M \in AC, N \in BC$ … $(MO) \equiv (ON)$ **(2p)**

În $\triangle CAB$, *CF* este mediană şi $MN \parallel AB \Rightarrow CF$ trece prin mijlocul lui (*MN*), adică prin *O*. Deci *F*, *O*, *C* sunt coliniare. **(2p)**